(Physics)

5 Electricity and Magnetism

5.1: Electrostatics

Coulomb's law: $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$

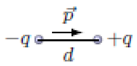
Electric field: $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$

Electrostatic energy: $U = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$

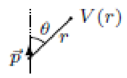
Electrostatic potential: $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

$$dV = -\vec{E}\cdot\vec{r}, \quad V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E}\cdot d\vec{r}$$

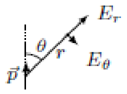
Electric dipole moment: $\vec{p} = q\vec{d}$



Potential of a dipole: $V = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$



Field of a dipole:



$$E_r = \frac{1}{4\pi\epsilon_0}\frac{2p\cos\theta}{r^3}, \quad E_\theta = \frac{1}{4\pi\epsilon_0}\frac{p\sin\theta}{r^3}$$

Torque on a dipole placed in $\vec{E}$: $\vec{\tau} = \vec{p}\times\vec{E}$

Pot. energy of a dipole placed in $\vec{E}$: $U = -\vec{p}\cdot\vec{E}$

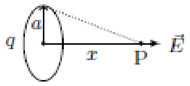
5.2: Gauss's Law and its Applications

Electric flux: $\phi = \oint \vec{E}\cdot d\vec{S}$

Gauss's law: $\oint \vec{E}\cdot d\vec{S} = q_{in}/\epsilon_0$

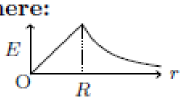
Field of a uniformly charged ring on its axis:

$$E_P = \frac{1}{4\pi\epsilon_0}\frac{qx}{(a^2+x^2)^{3/2}}$$

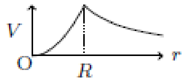


E and V of a uniformly charged sphere:

$$E = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}, & \text{for } r < R \\ \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}, & \text{for } r \geq R \end{cases}$$

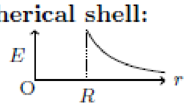


$$V = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{Qr^2}{R^3}, & \text{for } r < R \\ \frac{1}{4\pi\epsilon_0}\frac{Q}{r}, & \text{for } r \geq R \end{cases}$$

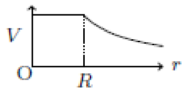


E and V of a uniformly charged spherical shell:

$$E = \begin{cases} 0, & \text{for } r < R \\ \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}, & \text{for } r \geq R \end{cases}$$



$$V = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{Q}{R}, & \text{for } r < R \\ \frac{1}{4\pi\epsilon_0}\frac{Q}{r}, & \text{for } r \geq R \end{cases}$$



Field of a line charge: $E = \frac{\lambda}{2\pi\epsilon_0 r}$

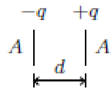
Field of an infinite sheet: $E = \frac{\sigma}{2\epsilon_0}$

Field in the vicinity of conducting surface: $E = \frac{\sigma}{\epsilon_0}$

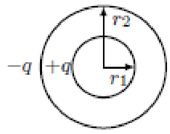
5.3: Capacitors

Capacitance: $C = q/V$

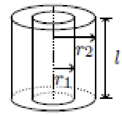
Parallel plate capacitor: $C = \epsilon_0 A/d$



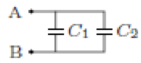
Spherical capacitor: $C = \frac{4\pi\epsilon_0 r_1 r_2}{r_2 - r_1}$



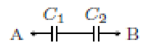
Cylindrical capacitor: $C = \frac{2\pi\epsilon_0 l}{\ln(r_2/r_1)}$



Capacitors in parallel: $C_{eq} = C_1 + C_2$



Capacitors in series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$



Force between plates of a parallel plate capacitor: $F = \frac{Q^2}{2A\epsilon_0}$

Energy stored in capacitor: $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$

Energy density in electric field E: $U/V = \frac{1}{2}\epsilon_0 E^2$

Capacitor with dielectric: $C = \frac{\epsilon_0 KA}{d}$

5.4: Current electricity

Current density: $j = i/A = \sigma E$

Drift speed: $v_d = \frac{1}{2}\frac{eE}{m}\tau = \frac{i}{neA}$

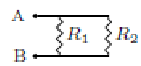
Resistance of a wire: $R = \rho l/A$, where $\rho = 1/\sigma$

Temp. dependence of resistance: $R = R_0(1 + \alpha\Delta T)$

Ohm's law: $V = iR$

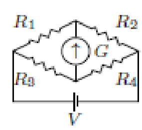
Kirchhoff's Laws: *(i) The Junction Law:* The algebraic sum of all the currents directed towards a node is zero i.e., $\Sigma_{node} I_i = 0$. (ii) *The Loop Law:* The algebraic sum of all the potential differences along a closed loop in a circuit is zero i.e., $\Sigma_{loop}\Delta V_i = 0$.

Resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$



Resistors in series: $R_{eq} = R_1 + R_2$

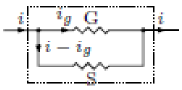
Wheatstone bridge:



Balanced if $R_1/R_2 = R_3/R_4$.

Electric Power: $P = V^2/R = I^2R = IV$

Galvanometer as an Ammeter:

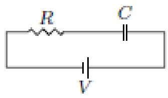


$$i_g G = (i - i_g)S$$

Galvanometer as a Voltmeter:

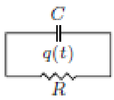
$$V_{\mathrm{AB}} = i_g(R + G)$$

Charging of capacitors:



$$q(t) = CV\left[1 - e^{-\frac{t}{RC}}\right]$$

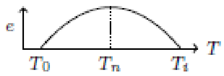
Discharging of capacitors: $q(t) = q_0 e^{-\frac{t}{RC}}$



Time constant in RC circuit: $\tau = RC$

Peltier effect: emf $e = \frac{\Delta H}{\Delta Q} = \frac{\text{Peltier heat}}{\text{charge transferred}}$.

Seeback effect:



1. Thermo-emf: $e = aT + \frac{1}{2}bT^2$
2. Thermoelectric power: $\mathrm{d}e/\mathrm{d}t = a + bT$.
3. Neutral temp.: $T_n = -a/b$.
4. Inversion temp.: $T_i = -2a/b$.

Thomson effect: emf $e = \frac{\Delta H}{\Delta Q} = \frac{\text{Thomson heat}}{\text{charge transferred}} = \sigma \Delta T$.

Faraday's law of electrolysis: The mass deposited is

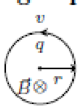
$$m = Zit = \frac{1}{F}Eit$$

where i is current, t is time, Z is electrochemical equivalent, E is chemical equivalent, and $F = 96485$ C/g is Faraday constant.

5.5: Magnetism

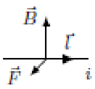
Lorentz force on a moving charge: $\vec{F} = q\vec{v} \times \vec{B} + q\vec{E}$

Charged particle in a uniform magnetic field:



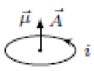
$r = \frac{mv}{qB}$, $T = \frac{2\pi m}{qB}$

Force on a current carrying wire:



$$\vec{F} = i\,\vec{l} \times \vec{B}$$

Magnetic moment of a current loop (dipole):

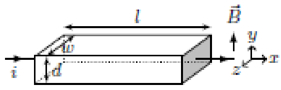


$$\vec{\mu} = i\vec{A}$$

Torque on a magnetic dipole placed in $\vec{B}$: $\vec{\tau} = \vec{\mu} \times \vec{B}$

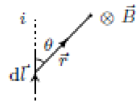
Energy of a magnetic dipole placed in $\vec{B}$:
$U = -\vec{\mu} \cdot \vec{B}$

Hall effect: $V_w = \frac{Bi}{ned}$

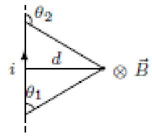


5.6: Magnetic Field due to Current

Biot-Savart law: $\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,\mathrm{d}\vec{l} \times \vec{r}}{r^3}$



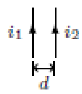
Field due to a straight conductor:



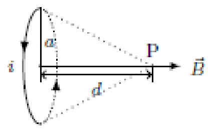
$$B = \frac{\mu_0 i}{4\pi d}(\cos\theta_1 - \cos\theta_2)$$

Field due to an infinite straight wire: $B = \frac{\mu_0 i}{2\pi d}$

Force between parallel wires: $\frac{\mathrm{d}F}{\mathrm{d}l} = \frac{\mu_0 i_1 i_2}{2\pi d}$

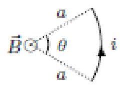


Field on the axis of a ring:



$$B_P = \frac{\mu_0 i a^2}{2(a^2 + d^2)^{3/2}}$$

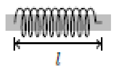
Field at the centre of an arc: $B = \frac{\mu_0 i \theta}{4\pi a}$



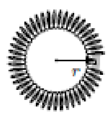
Field at the centre of a ring: $B = \frac{\mu_0 i}{2a}$

Ampere's law: $\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I_{\mathrm{in}}$

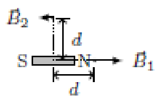
Field inside a solenoid: $B = \mu_0 n i$, $n = \frac{N}{l}$



Field inside a toroid: $B = \frac{\mu_0 N i}{2\pi r}$

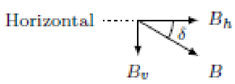


Field of a bar magnet:



$$B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d^3}, \quad B_2 = \frac{\mu_0}{4\pi}\frac{M}{d^3}$$

Angle of dip: $B_h = B\cos\delta$



Tangent galvanometer: $B_h \tan\theta = \frac{\mu_0 n i}{2r}$, $\quad i = K\tan\theta$

Moving coil galvanometer: $niAB = k\theta$, $\quad i = \frac{k}{nAB}\theta$

Time period of magnetometer: $T = 2\pi\sqrt{\frac{I}{MB_h}}$

Permeability: $\vec{B} = \mu\vec{H}$

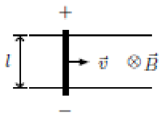
5.7: Electromagnetic Induction

Magnetic flux: $\phi = \oint \vec{B} \cdot \mathrm{d}\vec{S}$

Faraday's law: $e = -\frac{\mathrm{d}\phi}{\mathrm{d}t}$

Lenz's Law: Induced current create a B-field that opposes the change in magnetic flux.

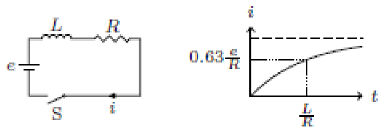
Motional emf: $e = Blv$



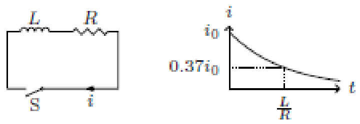
Self inductance: $\phi = Li, \quad e = -L\frac{\mathrm{d}i}{\mathrm{d}t}$

Self inductance of a solenoid: $L = \mu_0 n^2(\pi r^2 l)$

Growth of current in LR circuit: $i = \frac{e}{R}\left[1 - e^{-\frac{t}{L/R}}\right]$



Decay of current in LR circuit: $i = i_0 e^{-\frac{t}{L/R}}$



Time constant of LR circuit: $\tau = L/R$

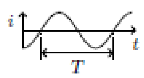
Energy stored in an inductor: $U = \frac{1}{2}Li^2$

Energy density of B field: $u = \frac{U}{V} = \frac{B^2}{2\mu_0}$

Mutual inductance: $\phi = Mi, \quad e = -M\frac{\mathrm{d}i}{\mathrm{d}t}$

EMF induced in a rotating coil: $e = NAB\omega \sin\omega t$

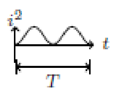
Alternating current:



$i = i_0 \sin(\omega t + \phi), \quad T = 2\pi/\omega$

Average current in AC: $\bar{i} = \frac{1}{T}\int_0^T i\,\mathrm{d}t = 0$

RMS current: $i_{\mathrm{rms}} = \left[\frac{1}{T}\int_0^T i^2\,\mathrm{d}t\right]^{1/2} = \frac{i_0}{\sqrt{2}}$



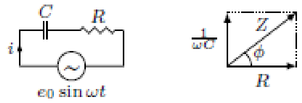
Energy: $E = {i_{\mathrm{rms}}}^2 RT$

Capacitive reactance: $X_c = \frac{1}{\omega C}$

Inductive reactance: $X_L = \omega L$

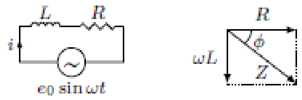
Imepedance: $Z = e_0/i_0$

RC circuit:



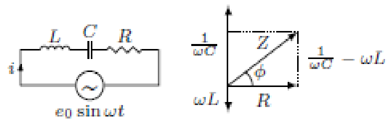
$Z = \sqrt{R^2 + (1/\omega C)^2}, \quad \tan\phi = \frac{1}{\omega CR}$

LR circuit:



$Z = \sqrt{R^2 + \omega^2 L^2}, \quad \tan\phi = \frac{\omega L}{R}$

LCR Circuit:

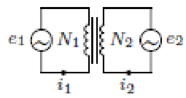


$Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}, \quad \tan\phi = \frac{\frac{1}{\omega C} - \omega L}{R}$

$\nu_{\mathrm{resonance}} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

Power factor: $P = e_{rms} i_{rms} \cos\phi$

Transformer: $\frac{N_1}{N_2} = \frac{e_1}{e_2}, \; e_1 i_1 = e_2 i_2$



Speed of the EM waves in vacuum: $c = 1/\sqrt{\mu_0 \epsilon_0}$